Visually discerning the curvature of the Earth

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Reports and photographs claiming that visual observers can detect the curvature of the Earth from high mountains or high-flying commercial aircraft are investigated. Visual daytime observations show that the minimum altitude at which curvature of the horizon can be detected is at or slightly below 35,000 ft, providing that the field of view is wide (60°) and nearly cloud free. The high-elevation horizon is almost as sharp as the sea-level horizon, but its contrast is less than 10% that of the sea-level horizon. Photographs purporting to show the curvature of the Earth are always suspect because virtually all camera lenses project an image that suffers from barrel distortion. To accurately assess curvature from a photograph, the horizon must be placed precisely in the center of the image, i.e., on the optical axis.

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1. Introduction

The health of the eye seems to demand a horizon. We are never tired, so long as we can see far enough.— Ralph Waldo Emerson [1]

The first direct visual detection of the curvature of the horizon has been widely attributed to Auguste Piccard and Paul Kipfer on 27 May 1931 [2]. They reported seeing it from a hydrogen-filled balloon at an elevation of 15,787 m (51,783 ft) over Germany and Austria. On 11 November 1935, Albert W. Stevens and Orville A. Anderson became the first people to photograph the curvature [3]. They were flying in the helium-filled Explorer II balloon during a record-breaking flight to an altitude of 22,066 m (72,395 ft) over South Dakota. Other claims have been made as to being the first to see the curvature of the Earth, but they seem to have come long after visual curvature had been established [4].

Since that time, countless people have claimed to be able to discern the curvature of the Earth as an upwardly arched horizon from high mountains or commercial aircraft. Some claim to see it from sea level or relatively low elevations [5]. We know that if we get high enough (i.e., from space), the curvature of the Earth is evident, but commercial aircraft seldom exceed altitudes of 40,000 ft (1 ft = 0.3048 m).

Interviews with pilots and high-elevation travelers revealed that few if any could detect curvature below about 50,000 ft. High-altitude physicist and experienced sky observer David Gutierrez [6] reported that as his B-57 ascends, the curvature of the horizon does not become readily sensible until about 50,000 ft and that at 60,000 ft the curvature is obvious. Having talked to many other high fliers (SR-71, U2, etc.), Gutierrez confirms that his sense of the curvature is the same as theirs. Passengers on the Concorde (60,000 ft) routinely marveled at the curvature of the Earth. Gutierrez believes that if the field of view (FOV) is wide enough, it might be possible to detect curvature from lower altitudes. The author has also talked to many commercial pilots, and they report that from elevations around 35,000 ft, they cannot see the curvature.

When trying to understand the perception of a curved horizon, two issues must be kept in mind. First, a large fraction of people wear eye glasses. Eye glasses produce a variety of distortions when the observer is not looking through the center part of the lens. Second, above the altitude of Mt. Everest, no observer can look directly at the horizon—he must look through a window or canopy. Plane-parallel windows like those on most aircraft will

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not render a flat horizon curved, but a curved window or canopy will.

What do we mean by "horizon"? Usually we mean the apparent boundary between the sea and sky, or distant landscape and sky. But Bohren and Fraser [7] showed that an observer at an altitude greater than a mile or so cannot see the hard Earth's horizon, i.e. the line-of-sight tangent point. Rayleigh scattering and scattering by aerosols usually reduce the height to less than a mile. The apparent horizon from commercial altitudes is not a sharp line, but rather a low-contrast transition from bright sky above to a slightly darker "sky" below. The location of this boundary is difficult to define (Fig. 1.)

Comparing the two images in Fig. 1 reveals a curious contrast reversal. The sky is generally brighter above the horizon at sea level, but darker above the horizon at high elevation. From sea level most of the atmosphere is above us, and so we see a lot of scattered light. From a high elevation, most of the atmosphere is below us, and a darker sky results. The contrast reversal is further accentuated by the sea's being relatively dark, while from high elevation the air and clouds below the horizon are relatively bright. Figure 2 shows vertical scans through the two images. The brightness differences are obvious, as is the amplitude of the brightness changes at the two horizons. The brightness change for the high-elevation horizon is less than 10% that of the sea-level horizon.

What the observer perceives as the horizon is actually a transition from an optically thick line of sight through atmosphere below the "horizon" to an optically thin line of sight above the "horizon." This apparent horizon is produced entirely within the atmosphere, and the hard Earth plays little or no role in its formation. The actual location probably corresponds to a line of sight with an optical depth near unity, which passes several miles above the surface of the Earth. From space this elevation is about 12 miles (see Appendix A; 1 mile = 1.609 km). Twelve miles (19 km) is about 0.3% of the Earth's radius, too small to influence the curvature discussed in this study.

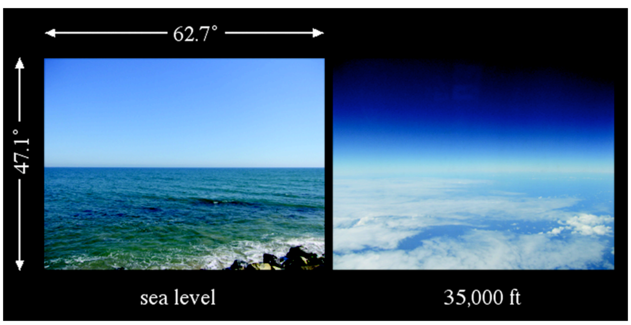

Fig. 1. The horizon from sea level (left) and from an elevation of 35,000 ft (right). Note the sharpness of the sea-level horizon and how indistinct the horizon from high elevation is. Also, note the overall contrast reversal between the two images. From sea level the sky is bright and the water is dark. But from high elevation the sky is dark and the sea and clouds are bright.

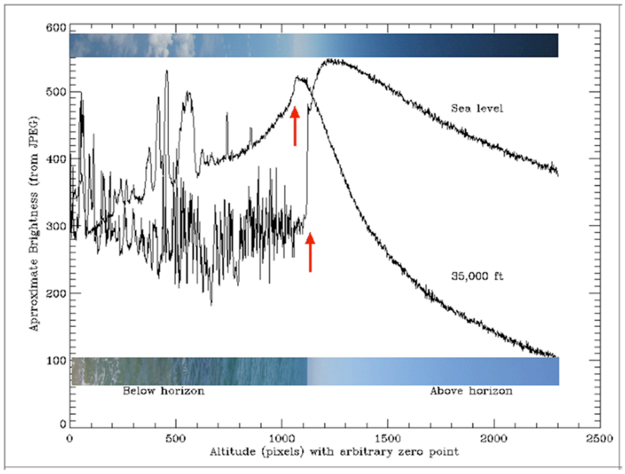

Fig. 2. (Color online) Vertical scans through the two images from Fig. 1. Also shown is a small strip from each image placed to match the scans. The sea-level horizon is sharp and has a high contrast. The transition from sea to sky is only about two pixels wide, corresponding to about 2 arc min. The angular resolution of the perfect eye is about 1 arc min, so virtually every naked-eye observer will see the horizon as absolutely sharp. The high-elevation horizon is almost as sharp, but its contrast is low, less than 10% of the sea-level horizon. The sharpness of the high-elevation horizon is a surprise in view of the fact that it is formed entirely within the atmosphere and is not a hard edge like the sea level horizon. The scans were made from JPEG images, and such images contain some compression. The signal level is only relative and, though not of photometric quality, is nonetheless monotonic and nearly linear with scene brightness.

Another potentially relevant issue in modeling the horizon's curvature is the distance to the horizon. As French [8] showed, refraction changes the distance to the horizon by a small amount relative to its geometric distance. Refraction may displace the location of the optical-depth-unity position in the sky, but it plays no role in the actual angular curvature of the horizon for two reasons: (1) we are not concerned with the altitude (or zenith distance) of the horizon, and (2) from high altitude, the hard horizon that French discusses cannot be seen [7].

Reports of curvature from high mountains and commercial jets are often supported with photographs showing the putative curvature [5]. Such photographs are suspect, as Figs. 3 and 4 show. Here I photographed the horizon from an elevation of about 8 ft (essentially sea level) using a Canon S70 PowerShot digital camera, like many commonly available cameras. The horizon was placed near the top, bottom, and center of the image. The barrel distortion is evident: lines above the center of the picture are arched upward (anticlinally) and those below center are arched downward (synclinally). Barrel distortion occurs when the pupil is placed away from the lens, a common technique used in camera lens manufacturing to produce a flat field. In view of the ability to make the horizon curved both upward and downward, and with the tendency of casual photographers to place

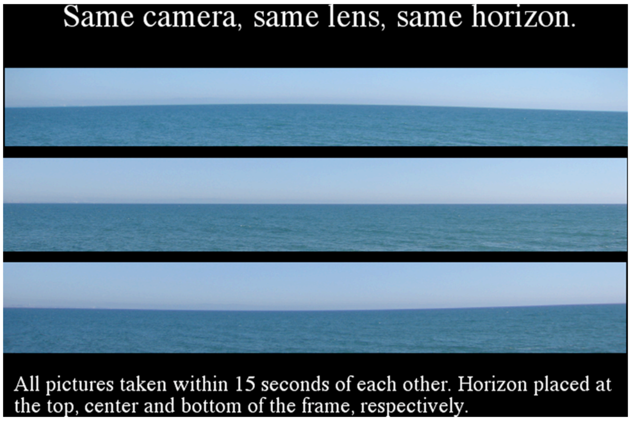

Fig. 3. (Color online) Apparent curvature of the horizon. Top, horizon placed near the top of the frame; middle, horizon placed in the center of the frame; bottom, horizon placed near the bottom of the frame. The apparent curvature is due to barrel distortion. These three images are horizontally compressed in Fig. 4 to enhance the visibility of the barrel distortion.

the horizon near the top of the image, where it appears curved upward like it would appear from very high elevation, we can dismiss most of the purported photographs of the curvature of the Earth as barrel distortion.

In this paper I investigate claims that the eye can detect the curvature of the horizon by using geometrical optics, simple models of the eye, and known perception effects. While the curvature of the Earth has been known since ancient time based on sailboat disappearances over the horizon and the shadow of the Earth cast onto the Moon during a lunar eclipse, I will not discuss these matters. Detecting the curvature of the horizon directly is a complex issue, and it is further aggravated by psychological factors: much has been written about the evocative effects of seeing the curvature of the Earth from space. People hope and often expect to see it and so they do, whether they actually do or not.

Fig. 4. (Color online) Apparent curvature of the horizon. On the left, the full frames are shown. On the right are the horizon photos cropped and compressed 10 : 1 horizontally to enhance the barrel distortion.

2. Visually Detect the Earth's Curvature

I first examined the horizon from commercial jet aircraft. While there was a general sense of the horizon, actually identifying the horizon's location was difficult. It was a very low-contrast boundary in a region of the sky where there were much higher-contrast changes. There were almost always clouds on the horizon that prevented accurate horizon location.

When the horizon was clear, detecting curvature from around $35,000$ ft was relatively easy, providing that a wide, unobstructed FOV was available. With a horizontal FOV of 90° or more, the curvature was subtle but unmistakable. Under similar conditions with a FOV smaller than about 60°, the curvature was not discernible. Thus, visually detecting the curvature would seem to depend on both the actual curvature and the FOV.

It seems likely that the curvature can be detected at elevations lower than $35,000$ ft, thus opening the door to the possibility of seeing it from high mountains. Mountaintops have very wide FOVs and thus may afford better viewing opportunities than aircraft. I regularly visit Mauna Kea (elevation $13,796\,\mathrm{ft} = 4205\,\mathrm{m}$) and Haleakala (elevation $10,223\,\mathrm{ft} = 3116\,\mathrm{m}$). From here, a relatively unobstructed horizon is visible in several directions. I was unable to convince myself that I could detect horizon curvature. Thus the altitude necessary to visually detect curvature would seem to be between about 14,000 and $35,000$ ft.

3. Photographing and Measuring the Curvature

Figure 5 shows a photograph taken over the Pacific Ocean somewhere between Los Angeles, California, and Kahului, Maui. The image is 62.7° × 47.1°. Care was taken to place the perceived horizon precisely in the center of the frame (plus or minus a few pixels, or about 3 arc min). As noted earlier, visual observations found that the curvature was subtle but discernible. The image was imported into a drawing program, and three small dots were placed on the horizon: one each at the left and right edge of the image, and one near the center. A line was drawn between the left and right dots and was found to fall slightly below the center dot, a clear indication of curvature (inset in Fig. 5). The measured distance (sagitta) was 0.51°, or about 17 pixels (note that the

Fig. 5. (Color online) This picture shows a photograph of the horizon from an elevation of 35,000 ft and with a horizontal FOV of 62.7°. Also shown are the three reference points defining the horizon, a horizontal line connect the left- and right-hand points, and the measured amount of sagitta (see inset for a closeup of the sagitta measurement).

horizontal angle from the limb center is half of the FOV, or about 31.3°).

4. Model of the Earth's Curvature

To interpret the measurements above, a simple geometrical model of the viewing conditions was constructed. Figure 6 shows an observer's view of the Earth (radius R) from an arbitrary elevation h above the surface. The amount by which the apparent Earth limb falls below the horizon S as a function of horizontal distance x from the vertical is apparent by inspection:

$$S = R - (R^2 - x^2)^{1/2}. \quad (1)$$

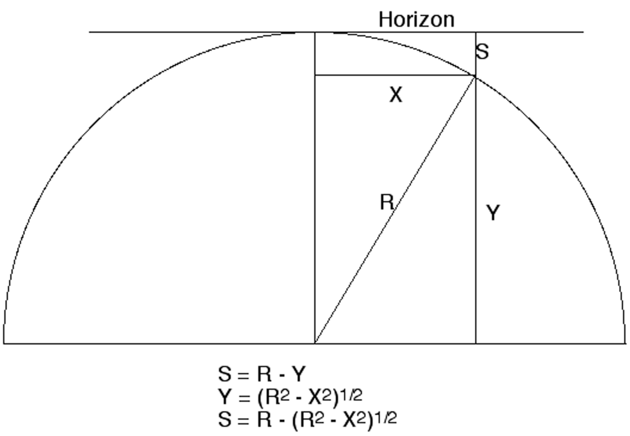

Fig. 6. Model of the horizon and the Earth's curvature as seen by an observer from an arbitrary elevation h above the surface. The amount S (sagitta) by which the apparent Earth limb falls below the horizon is easily calculable: $S = R - (R^2 - X^2)^{1/2}$. To convert this linear dimension to an angular dimension, we need only divide each quantity by the distance to the horizon $D \approx (2Rh + h^2)^{1/2}$).

To convert from linear dimensions to angular dimensions, we need only divide each quantity by the distance to the horizon:

$$D \approx (2Rh + h^2)^{1/2}. \quad (2)$$

I assumed that the distance to the horizon was the same at every azimuth, i.e., at every horizon point in the image. I then calculated a family of limb curvatures for various observer elevations.

Figure 7 shows the family of curvatures as a function of azimuth with observer height as a parameter. The asterisk shows the measurement discussed above for $h = 35{,}000$ ft, a horizontal FOV of 62.7° (2×) and observed $S = 0.51°$. The measurement agrees well with the theoretical expectations. Several other measurements from slightly different elevations were made, and the findings were consistent. In an attempt to further populate Fig. 7 with observations from significantly higher and lower elevations, I asked the pilots to change elevations; my requests were ignored.

5. Summary and Conclusions

In view of the agreement between the visual observations, measurements of the photographs, and the theoretical curvatures, it seems well established that the curvature of the Earth is reasonably well understood and can be measured from photographs. The threshold elevation for detecting curvature would seem to be somewhat less than 35,000 ft but not as low as 14,000 ft. Photographically, curvature may be measurable as low as 20,000 ft.

Appendix A

The estimate of a 12 mile elevation for the apparent horizon discerned from high altitude that was mentioned Section 1 is based on the following argument. The vertical optical depth from sea level to space at 0.55 μm is about 0.13, corresponding to 1 air mass [9]

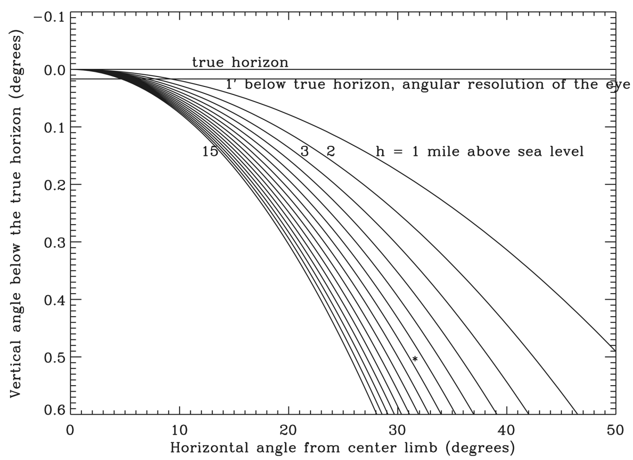

Fig. 7. Model calculations of the curvature of the Earth from various elevations. The curve for an elevation of 35,000 ft is the thick line, and the asterisk shows the measurement from Fig. 5.

The horizon air mass for a sea-level observer is about 38, or 76 for a passage through the entire atmosphere tangential to the surface. This corresponds to an optical depth of $76 \times 0.13 = 9.88$. From space an observer would find the apparent horizon (limb) at an optical depth of approximately unity along that tangential line of site. Since the atmosphere is exponential with height, we can estimate the elevation E of the horizon seen by a high-altitude observer in space by scaling the optical depth to unity, using the density scale height of the atmosphere, i.e.,

$$9.88\ \exp(-E/\mathrm{ho}) = 1.0,$$

where ho is the scale height of the atmosphere, taken here as 8.4 km (5.2 miles). Solving the above equation for E yields 19.2 km, or 11.9 miles.

I thank David Gutierrez, Lawrence S. Bernstein, Andrew Young, and William Livingston for many useful discussions about the horizon.

References and Notes

1. R. W. Emerson, *Nature: Addresses, and Lectures*, new and revised ed. (Houghton, Mifflin, 1884), p. 22..
2. Piccard is widely believed to be the first. There are many references to his achievement on the Internet, most of them certainly derivative. I contacted the Piccard family and they were aware of the claim but had no hard evidence or literature citation backing it up.
3. S. W. Bilsing and O. W. Caldwell "Scientific events," Science **82** 586–587 (1935).
4. A brass plaque placed at the Lamont Odett vista point in Palmdale, Calif., by E. Vitus Clampus claims that X-1A pilot Arthur "Kitt" Murray was the first person to see the curvature of the Earth. The plaque does not cite the year or altitude, but, according to the NASA archives, it was probably on 26 August 1954 when Murray took the X-1A to a record-breaking altitude of 90,440 ft (27,566 m).
5. Entering "curvature of the earth" into the image search using any search engine will find thousands of images. Most photographers place the horizon near the top of the frame in order to capture the scene of interest below the horizon. The resulting barrel distortion produces a pronounced upward (anticlinal) curvature of the horizon that most people incorrectly interpret as the curvature of the Earth.
6. D. Gutierrez, djgutierrez1@verizon.net (personal communication, 2007).
7. C. F. Bohren and A. B. Fraser, "At what altitude does the horizon cease to be visible?" Am. J. Phys. **54**, 222–227 (1986).
8. A. P. French, "How far away is the horizon?" Am. J. Phys. Vol. **50**, 795–799 (1982).
9. E. J. McCartney, *Optics of the Atmosphere* (Wiley, 1976), Fig, 4.8, p 205.